LECTURE NOTES
Sensitivity and Uncertainty Analysis

Learning objectives

1. **Describe** reasons to include uncertainty in PBL designs
2. **List** factors that effect reservoir firm yield
3. **Propagate** effects of uncertain parameters and factors
4. **Infer** expected reservoir firm yield from uncertain annual demand

1. Motivation

- Up to now in our PBL designs and reports, we have only qualitatively considered uncertainties
  - Identified sources
  - Hypothesized effects
- All deterministic designs (use fixed, specified values)
- It's time to explicitly (mathematically) include uncertainty!
- What are the key uncertainties that affect the firm yield of a reservoir?

2. What factors effect reservoir firm yield?

Recall that reservoir firm yield is affected by:

- Reservoir deliveries to demand sites
- Reservoir release rules
- Demand site delivery requirements
- Available water
  - Inflows
  - Evaporation
  - Losses
  - Storage in previous time step
  - Other considerations

Essentially, there is a function:

Firm yield = f(inflows, evaporation, losses, delivery requirements, release rules, …)

How do changes in one or more of the input parameters effect firm yield?

3. Propagate effects of uncertain parameters

A. General Approach

i. Identify parameters/variables that are known (certain)
ii. Identify unknown parameters/variables that are independent of one-another (uncorrelated)
iii. Identify unknown parameters/variables that depend on (or correlate to) one or more other unknown parameters/variables. (Specify functional relation).
iv. Mathematically propagate the uncertainties

Case 1. 1 or more unknown, independent parameters of interest but you do **not** know the probability distribution of the uncertain parameters.

a. Specify an appropriate range for each uncertain parameter.

b. Conduct sensitivity analysis (propagate results for select test values in the appropriate range)

c. Report results as a spider or tornado plot.

Example 1. How do uncertain Manning's N values affect flow in a channel if the uncertain N values vary +/- 0.01 about the base case value of 0.02? If the channel width varies +/- 1.5 ft above the base case value of 10 feet?

Answer 1. See Excel workbook.

Case 2. 1 or more unknown, independent parameters of interest and you **do** know the probability distribution of the uncertain parameters.

Use analytical, Monte-Carlo sampling, numerical integration, or simulation to propagate results.

For example with Monte Carlo simulation:

a. Random sample from the probability density function (pdf) for the parameter to generate a list of parameter values

b. Propagate the result of interest for each random sampled parameter value

c. Describe the statistical properties of the propagated results (e.g., expected value and variance).

Example 2. How do uncertain Manning's N values affect flow in a channel if the N values are normally distributed with a mean and standard deviation of 0.03 and 0.08?

Answer 2. See Excel workbook.

4. Infer expected reservoir firm yield from uncertain delivery targets

Example 3. How do uncertain delivery targets to the New Cache Valley users effect the firm yield of the Above Cutler Reservoir? What steps should you undertake in WEAP to complete this analysis?

Criteria to Select the Appropriate Uncertainty/Propagation Method

Uncertainty Method	Conditions to Use
Sensitivity analysis	• Pdfs are unknown for parameters of interest
Analytical	• All unknown parameters are independent • Have simple, tractable pdfs • Can integrate pdfs • Easiest for one single unknown
Monte-Carlo sample	• Samples are independent • Unknowns within a sample either independent or dependent on other independent unknowns within the same sample • Can invert cdfs for independent unknowns to get the variable value as a function of a random number between 0 and 1
Numerically integrate	• Cannot analytically integrate the pdf or invert the cdf for at least one unknown independent variable • Easiest when there is only one unknown independent variable
Simulate	• Sequential order is important (for example, temporal correlation among unknowns) • Complex dependencies among unknowns

5. Wrap-up