LECTURE NOTES
Engineering Economics (Recall + Extension)

Learning objectives

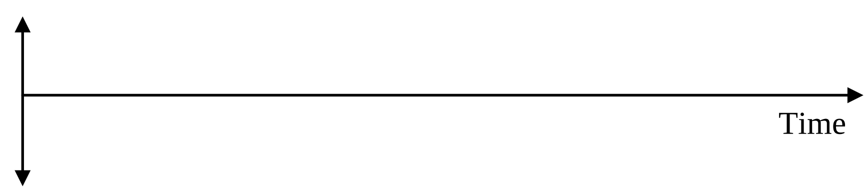
1. **Recall** definitions of cash flow stream, interest rate, time horizon
2. **Calculate** financial performance indicators
   A. Present value (PV)
   B. Net present value (NPV)
   C. Internal rate of return (IRR)
   D. Benefit-cost ratio (BCR)
   E. Payback period
3. **Apply** performance indicators to make decisions
4. **Describe** limitations of performance indicators

1. Recall definitions

A. Cash flow stream – the activity receipts (benefits) and disbursements (costs) at different points in time.

Conventions: Benefits are listed as positive; costs as negative. We'll denote the cash flow stream x_t (at the end of periods t=0, 1, 2, … n) so that x_1 = \$15 and x_2 =-\$20 would be a \$15 receipt and \$20 disbursement at the end of years 1and 2.

EXAMPLE 1. Draw the cash flow stream for a homeowner who purchases and installs a new, more efficient irrigation system. The system costs \$500, reduces water bills by \$25/year, requires \$50 of upkeep at the end of every even year, and has an expected life of 10 years?



B. Interest rate – the time value of money. The mathematical way to relate the value of \$1 today (now) to its value in the future (hereafter, *i* in the notes).

Units of *i* =

Figure 1 shows how national interest rates have changed over time.

C. Time horizon – the period of time [generally years] considered by the financial analysis.

This (planning) horizon must be identical across all alternatives considered!

What is the time horizon you will use for PBL-1??

2. Calculate financial performance indicators

A. Present value (PV): turns a future value into its equivalent present worth. Also called discounting.

$PV = x_t(P/F, i, t) = x_t(1+i)^{-t}$ (simple compounding)

$PV = x_t e^{-rt}$ (continuous compounding; r = nominal rate)

See Pages X and Y for common formulas that use the interest rate to discount.

EXAMPLE 2. What is the present value of the 10 annual water bill reductions in Example 1 with an interest rate of 3%? 8%?

PV = present value of all reduced water bills

$$PV = x_t(P/A, i, 10) = \$25\frac{(1+i)^{10}-1}{i(1+i)^{10}}$$

PV = $213 (at 3% interest)
PV = $168 (at 8% interest)

B. Net present value (NPV): The present value of an entire cash flow stream.

EXAMPLE 3. What is the net present value of the Example 1 project with an interest rate of 3%? 8%?

Remember, we ignore past (sunk) costs or benefits in the analysis. Why?

What are some example sunk costs in Example 1?

C. Internal rate of return (IRR): the interest rate that sets NPV to zero.

IRR = i when $NPV(i,x_t) = 0$.

1. Set up an equation that relates NPV to the interest rate i.
2. Solve for several interest rates
3. Graph NPV versus *i* and see where NPV crosses zero. *or*
4. In Excel, use the Solver Add-in to find the *i* that makes NPV zero.

D. Benefit-Cost ratio (BCR): is the present value of benefits divided by the present value of costs.

$$BCR = \frac{PV(Benefits)}{PV(Costs)}$$

EXAMPLE 4. What is the benefit-cost ratio for the Example 1 project under the two interest rates?

Solution: See Excel Example.

E. Payback period (PP): the time required to recover the investment costs.
Simple payback period (SPP): the payback period that ignores interest.

PP = n when $\sum_{t=0}^{n} NPV(i, x_t) = 0$

If there is only an initial cost and subsequent constant benefits:

$$SPP = \frac{x_0}{x_1}$$

EXAMPLE 4. In VLE Example #1 (from September 8), what is the simple payback period for replacing cool- with warm-season turf?

3. Decision Criteria

If there are many potential projects (or alternatives), which one should we select?

Performance Indicator	**Select project when**	**Reject project when**
Net present value		
Internal rate of return		
Benefit-cost ratio		
Payback period		

4. Limitations of these Performance Indicators

A. Must identify and quantify project impacts. What conservation action impacts may be difficult to quantify?

B. Must identify the time horizon. What may cause the time horizon of a conservation action to be uncertain?

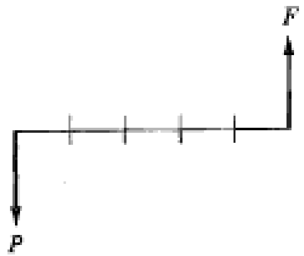
C. Must specify a discount factor (interest rate). What interest rate should your PBL-1 client use?

D. Multiple IRRs exist when the cash flow stream switches from benefits to costs

E. BCRs change when counting costs as negative benefits (or vice-versa).

   See Excel Example

Single Payment



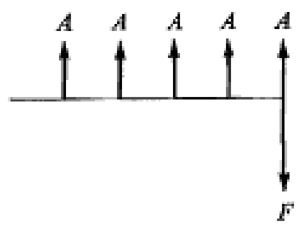
Compound Amount:

To Find F Given P $(F/P,i,n)$ $F = P(1+i)^n$

Present Worth:

To Find P Given F $(P/F,i,n)$ $P = F(1+i)^{-n}$

Uniform Series

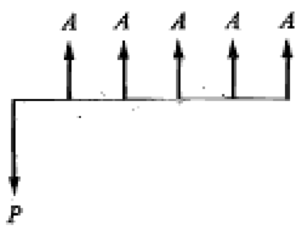


Series Compound Amount:

To Find F Given A $(F/A,i,n)$ $F = A\left[\frac{(1+i)^n - 1}{i}\right]$

Sinking Fund:

To Find A Given F $(A/F,i,n)$ $A = F\left[\frac{i}{(1+i)^n - 1}\right]$



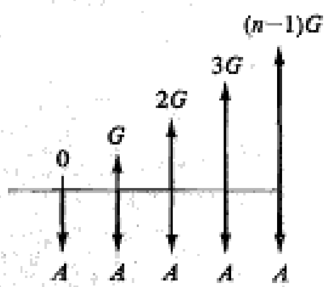
Capital Recovery:

To Find A Given P $(A/P,i,n)$ $A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$

Series Present Worth:

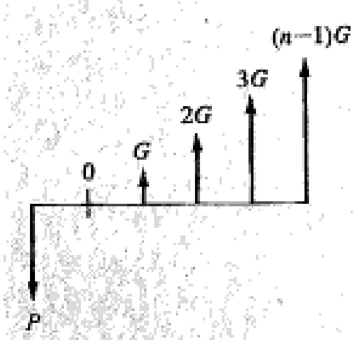
To Find P Given A $(P/A,i,n)$ $P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

Arithmetic Gradient



Arithmetic Gradient Uniform Series:

To Find A Given G $(A/G,i,n)$ $A = G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$

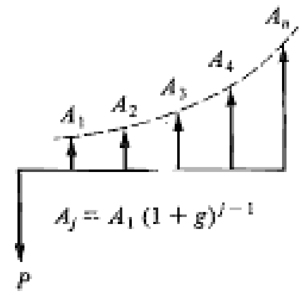


Arithmetic Gradient Present Worth:

To Find P Given G $(P/G,i,n)$ $P = G\left[\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right]$

Geometric Gradient

Geometric Series Present Worth:



To Find P Given A_1, g — $(P/A,g,i,n)$ When $i = g$ — $P = A_1[n(1+i)^{-1}]$

To Find P Given A_1, g — $(P/A,g,i,n)$ When $i \neq g$ — $P = A_1\left[\frac{1-(1+g)^n(1+i)^{-n}}{i-g}\right]$

Continuous Compounding at Nominal Rate r

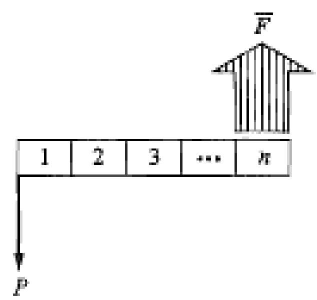
Single Payment: $F = P[e^{rn}]$ $P = F[e^{-rn}]$

Uniform Series: $A = F\left[\frac{e^r-1}{e^{rn}-1}\right]$ $A = P\left[\frac{e^{rn}(e^r-1)}{e^{rn}-1}\right]$

$F = A\left[\frac{e^{rn}-1}{e^r-1}\right]$ $P = A\left[\frac{e^{rn}-1}{e^{rn}(e^r-1)}\right]$

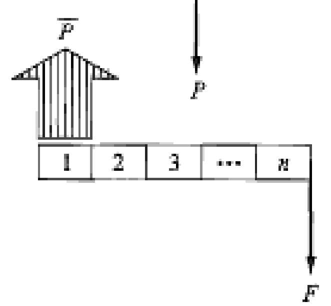
Continuous, Uniform Cash Flow (One Period) With Continuous Compounding at Nominal Rate r

Present Worth:



To Find P Given $\overline{F}$ — $(P/\overline{F},r,n)$ — $P = \overline{F}\left[\frac{e^r-1}{re^{rn}}\right]$

Compound Amount:



To Find F Given $\overline{P}$ — $(F/\overline{P},r,n)$ — $F = \overline{P}\left[\frac{(e^r-1)(e^{rn})}{re^r}\right]$

Compound Interest

i = Interest rate per interest period*.

n = Number of interest periods.

P = A present sum of money.

F = A future sum of money. The future sum F is an amount, n interest periods from the present, that is equivalent to P with interest rate i.

A = An end-of-period cash receipt or disbursement in a uniform series continuing for n periods, the entire series equivalent to P or F at interest rate i.

G = Uniform period-by-period increase or decrease in cash receipts or disbursements; the arithmetic gradient.

g = Uniform *rate* of cash flow increase or decrease from period to period; the geometric gradient.

r = Nominal interest rate per interest period*.

m = Number of compounding subperiods per period*.

$\overline{P}, \overline{F}$ = Amount of money flowing continuously and uniformly during one given period.

*Normally the interest period is one year, but it could be something else.