Hypothetical PBL-Opportunity: Period-of-Record Analysis

City Z plans to build a dam and reservoir just below the confluence of the right- and left-hand forks in the hypothetical lecture watershed (Figure 1) and has hired your firm to develop the period-of-record of <u>impaired</u> inflow to the reservoir. The city will use this period of record to design, size and develop an operating policy for the reservoir. Key related events in the watershed development were:

- 1921. City installed and started operating Gage A
- 1952. City first investigated dam site: installed and started operating Gage B (just below the upper confluence) and Gage C.
- 1968. A neighboring city started legally diverting an unknown quantity of water out of the basin from the left-hand fork upstream of Gage B.
- 1991. Gage A fell into disrepair.

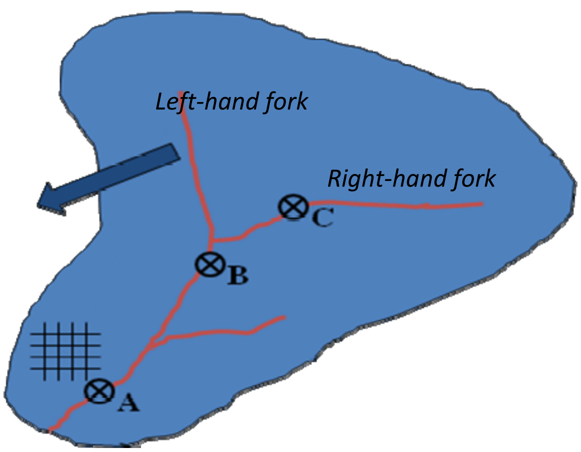

Figure 1. Schematic of Lecture Watershed

Period-of-Record of Impaired Flows at the Potential Dam Site in the Hypothetical Lecture Watershed

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PBL-Example Report #1

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Introduction

City Z plans to build a dam and reservoir just below the confluence of the right- and left-hand forks in the hypothetical lecture watershed. The gaged record of flow at the dam site spans just 40 years and the city desires a longer estimate of monthly impaired flows and flow variations to aid in reservoir design, sizing, and operations. This report summarizes the existing primary and secondary gaged flow data in the watershed and presents two sets of regressions that estimate impaired flow at the dam site over a longer period of time from the available secondary data. Discussion highlights uncertainties associated with the regressions used to extend the period-of-record for analysis.

Background

The hypothetical watershed covers 450 square miles; 315 square miles are upstream of the potential dam site (Figure 1). City Z installed Gage A in 1921. Later in 1952, the city installed Gage B (just downstream of the potential dam site) and Gage C (upstream of the dam site on the right-hand fork). In 1968, a neighboring city started legally diverting an unknown quantity of water out of the watershed upstream of Gage B on the left-hand fork. In 1991, Gage A fell into disrepair.

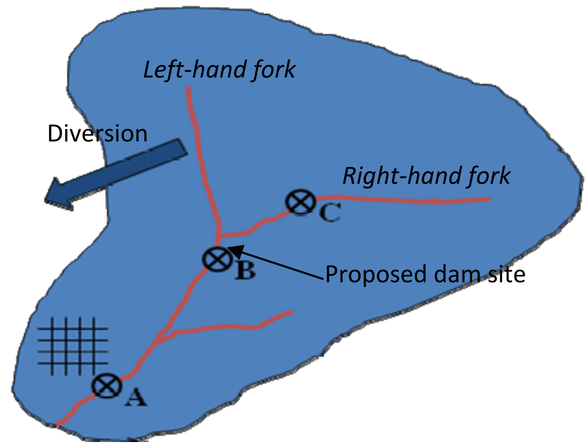

Figure 1. Hypothetical lecture watershed

Presently, the city has 41 years (1968 to 2008) of daily average flow measurements at Gage B that represent impaired inflow to the reservoir. Table 1 catalogues other secondary data of unimpaired and impaired flows at nearby locations which are used to extend the period-of-record of impaired inflow to the potential dam site.

Table 1. Existing secondary flow data in the hypothetical lecture watershed

Location	Measurement	Condition	Average flow (cfs)	Dates	Data source
A	Flow (daily average cfs)	Unimpaired	456	1921 – 1968	CUASHI (2008)
A	Flow (daily average cfs)	Impaired	123	1968 – 1991	CUASHI (2008)
B	Flow (daily average cfs)	Unimpaired	89	1952 – 1968	City Z (2008)
C	Flow (daily average cfs)	Unimpaired	56	1952 – 2008	City Z (2008)

Methods

We developed the period-of-record of impaired inflow to the reservoir by partitioning time into three intervals and developing conversions and regressions specific to the primary and secondary data available in each time interval. For 1968 to 2008, we summed daily average flow measurements in cfs over each month and multiply by a conversion factor to get a monthly flow volume in af. For the second time period from 1952 to 1968, we estimated the monthly diversion flow on the left-hand fork upstream of Gage B and subtract this estimated diverted amount from a converted monthly unimpaired flow observed at B (Appendix A). The diversion amount was estimated by: (a) linearly correlating unimpaired flows at Gages B and C using flow observations from 1952 to 1968. (b) Estimating unimpaired flows at Gage B from 1968 to present using the linear regression parameters and observations of unimpaired flow at Gage C over the same time period. Finally, we (c) subtracted the impaired flow observed at Gage B from the estimate of unimpaired flow at Gage B developed in step (b).

For the third time period from 1921 to 1951, we instead relied on unimpaired flows observed at Gages A and B (Appendix B). First, we used a power function regression to correlate unimpaired flows over the time period the two observations overlap (1952 to 1968). Then, we use unimpaired flows observed at Gage A between 1921 and 1951 and the power function regression coefficients and to estimate impaired flow at B. Finally, we subtract off the estimated diversion amount. Appendices A and B present regressions for the two time periods.

Results

The resulting period-of-record of monthly estimated and measured impaired inflows to the potential reservoir site spans from 1921 through 2008 (Appendix C). Impaired flows range from slightly above 100 to more than 1,600 af/month and show no seasonal behavior.

Discussion

Period-of-record analysis extended the existing 40-year record of impaired flows at Gage B (1968 to 2008) back to 1921 and more than doubled the length of hydrological record. However, there are several errors associated with the methods use to extend the period-of-record. First, the estimate of the upstream diversion amount is generally less than 500 af/month (Appendix A) and of similar magnitude to the tributary flows accumulated between Gages C and B and on the left-hand fork. This uncertainty makes it difficult to correct unimpaired flow measurements at Gage B. Second, comparing impaired flow estimates from the two regression approaches over the period 1952 to 1968 when both methods can be used shows good agreement but estimates differ by up to 100 af/month (Appendix C). Although correlation coefficients for both regressions are above 0.85, together, the two errors constitute a significant portion of impaired flow estimates at Gage B, especially for low flow conditions.

Conclusions

A period-of-record analysis used secondary records of impaired and unimpaired flows observed at 3 gages in the hypothetical lecture watershed to extend an existing 41-year record of impaired flows observed just below the confluence of the right- and left-hand forks back another 47 years to as early as 1921. The analysis relied on linear and power function regressions and monthly estimates of an upstream diversion out of the basin. Correlations are strong for the two regressions, but uncertainties in differentiating the diversion amount from the local contributions of inflow contribute significant error to the estimate of impaired flow at the confluence, particularly at low flow conditions. The period-of-record represents a time-series of impaired inflows to the reservoir (corrected for upstream diversions out of the watershed) that can be used for statistical analysis and to design, size, and plan operations for the dam proposed just below the confluence.

References

City Z. (2008). "Annual Report of River Gagging in the Hypothetical Lecture Basin." PR-10-2008. City Z, State Y.

Consortium of Universities for the Advancement of Hydrologic Science, Inc. (CUAHSI). "HydroSeek." September 30, 2008 <http://his.cuahsi.org/hydroseek.html>. Path: Site 1; Keywords: Hypothetical Lecture Basin.

Appendix A. Impaired flow estimation at Gage B from 1952 to 1968

This appendix describes the regression methods and manipulations used to estimate impaired flows at Gage B for 1952 to 1968 using unimpaired flows measured at Gage B from 1952 to 1968, unimpaired flows measured at Gage C from 1952 to 2008, and impaired flows measured at Gage B from 1968 to 2008. After 1968, a diversion upstream of Gage B withdrew an unknown quantity of water from the basin. This estimation method is used to quantify monthly diversion amounts and correct unimpaired flow measurements at Gage B for the period 1952 to 1968 as though the diversion were actually in place.

The regression and estimation method requires five steps.

Step 1. Aggregate daily average flow observations in cfs to monthly flow volumes in af at each gage by summing the daily average readings for the month and multiplying by a conversion factor of 1.98 af/(mon cfs/day).

Step 2. Linearly correlate monthly unimpaired flows at Gages B and C using flow observations from 1952 to 1968 (Figure A1). A least squares regression made in Excel and based on Equation A1,

$$\left(UIP\ \hat{f}\ at\ B\right) = \hat{\alpha} + \hat{\beta}\left(UIP\ f\ at\ C\right) \quad \text{(A1)}$$

where $\left(UIP\ \hat{f}\ at\ B\right)$ is the estimate of unimpaired monthly flow at Gage B [af/mon], (*UIP f at C*) is unimpaired monthly flow measured at Gage C [af/mon], $\hat{\alpha}$ is the estimated intercept [af/mon], and $\hat{\beta}$ is the estimated slope [unitless], finds the intercept = -608 af/mon and the slope = 5.5. Here and subsequently, parameters with the superscript "^" means the parameter is estimated while parameters without a superscript are measured. Both $\hat{\alpha}$ and $\hat{\beta}$ are empirical regression coefficients. The coefficient of correlation (r^2) for the regression is 0.93 and indicates the regression equation explains approximately 93% of the variation of unimpaired flows at the two gages. This strongly linear relation means the catchments upstream of Gages B and C proportionately contribute runoff across storm events of different runoff magnitudes.

Step 3. Apply Equation A1 to estimate unimpaired flows at Gage B from 1968 to present from observations of unimpaired flow at Gage C over the same time period.

Step 4. Subtract the estimate of unimpaired flow at Gage B made in Step 3 from impaired flow observed at Gage B over the same time period to estimate the diversion amount (Equation A2).

$$\hat{DIV} = \left(UIP\ \hat{f}\ at\ B\right) - \left(IP\ f\ at\ B\right) = \hat{\alpha} + \hat{\beta}\left(UIP\ f\ at\ C\right) - \left(IP\ f\ at\ B\right) \quad \text{(A2)}$$

Here, $\hat{DIV}$ = the estimated diversion amount [af/mon], (*IP f at B*) = impaired monthly flow measured at Gage B [af/mon], and $\left(UIP\ \hat{f}\ at\ B \right)$, (*UIP f at C*), $\hat{\alpha}$, and $\hat{\beta}$ are as defined previously.

Plotting the 41-year time series of estimated monthly diversion amounts shows diversions are generally less than 500 af/mon but change significantly from month to month (Figure A2). Table A1 summarizes the average and standard deviation of diversion amounts by month of the year. The diversion volume appears to almost double from May to June.

Step 5. Rearranging Equation A2 gives Equation A3 which estimates impaired flows at gage B $\left(IP\ \hat{f}\ at\ B \right)$ [af/month] as a function of the estimated monthly diversion amount ($\hat{DIV}(t)$) [af/month] and monthly unimpaired flow observed at gage B (*UIP f at B*) [af/mon].

$$\left(IP\ \hat{f}\ at\ B \right) = \left(UIP\ f\ at\ B \right) - \hat{DIV}(t) \quad \text{(A3)}$$

Here, the parameter *t* represents the month and indicates that the diversion amount varies by month of the year.

Table A1. Estimated diversion values (af/mon)

Month	Average	Standard Deviation
January	176.0	55.9
February	165.0	56.2
March	180.9	58.2
April	177.5	60.6
May	345.1	71.0
June	349.2	74.0
July	350.4	43.3
August	334.6	78.0
September	340.0	104.6
October	184.7	70.2
November	160.0	64.9
December	173.4	72.9

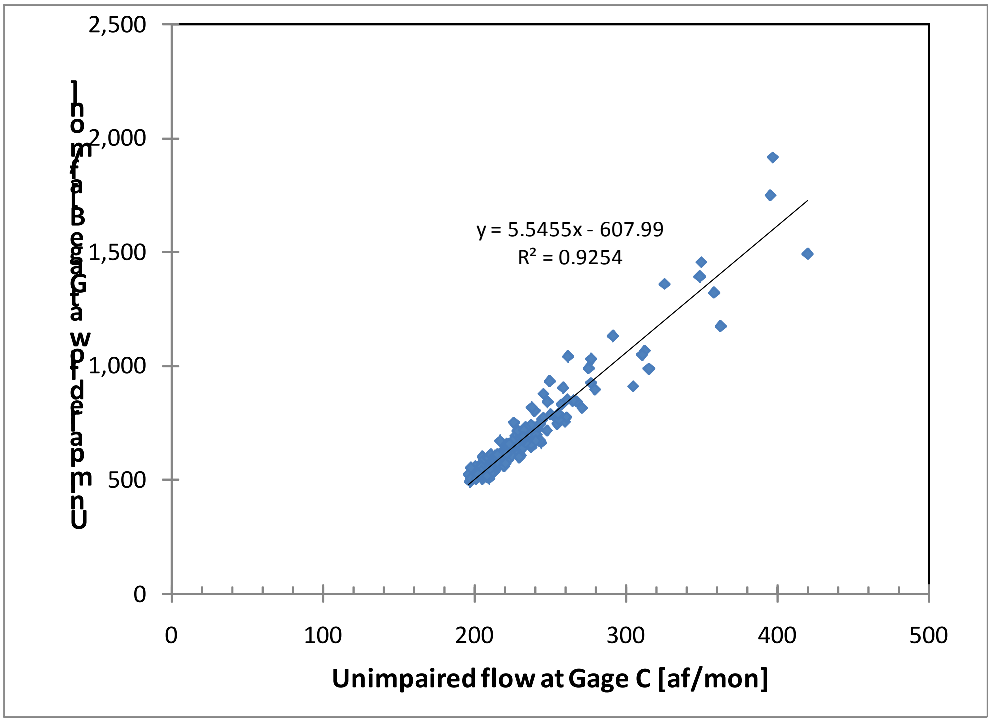

Figure A1. Correlation of unimpaired flows at Gages B and C from 1952 to 1968.

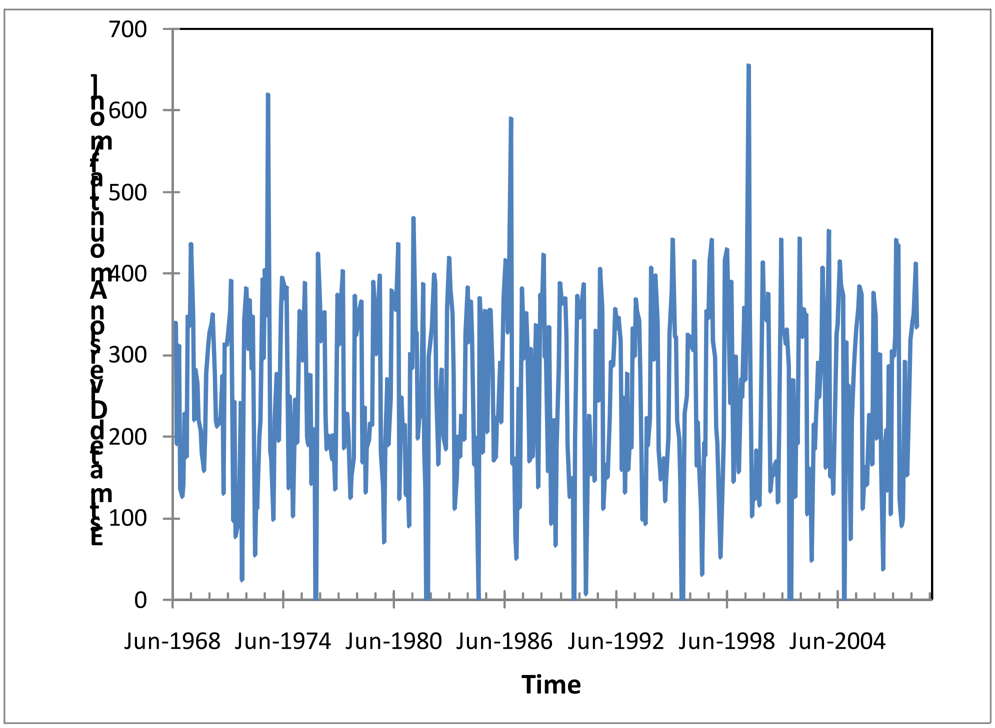

Figure A2. Time-series of estimated diversion withdrawals from 1968 to 2008

Appendix B. Impaired flow estimation at Gage B from 1921 to 1952

This appendix describes the regression methods and manipulations used to estimate impaired flows at Gage B for 1921 to 1952 using unimpaired flows measured at Gage B from 1952 to 1968, unimpaired flows measured at Gage A from 1921 to 1968, and diversion amounts estimated in Appendix A. The method estimates unimpaired flow at Gage B then corrects that estimate as though the upstream diversion were in place.

The regression and correction method requires four steps.

Step 1. Again, aggregate daily average flow observations in cfs to monthly flow volumes in af at each gage by summing the daily readings for the month and multiplying by a conversion factor of 1.98 af/(mon cfs/day).

Step 2. Use a power function to correlate unimpaired flows at Gage B to unimpaired flows at Gage A using flow observations from 1952 to 1968 (Figure B1). A least squares regression made in Excel and based on Equation B1,

$$\left(UIP\ \hat{f}\ at\ B\right) = \hat{\rho}\left(UIP\ f\ at\ A\right)^{\hat{\sigma}} \tag{B1}$$

where $\left(UIP\ \hat{f}\ at\ B\right)$ is the estimate of monthly unimpaired flow at Gage B [af/mon], (*UIP f at A*) is unimpaired flow measured at Gage A [af/mon], $\hat{\rho}$ is the estimated multiplier [unitless], and $\hat{\sigma}$ is the estimated exponent [unitless], finds the multiplier = 49.4 and the exponent = 0.36. Again, parameters with the superscript "^" means the parameter is estimated while parameters without a superscript are measured. Both $\hat{\rho}$ and $\hat{\sigma}$ are empirical regression coefficients. The coefficient of correlation (r^2) for the regression is 0.98 and indicates the regression equation explains approximately 98% of the variation of unimpaired flows at the two gages. The estimate of the regression exponent is less than 1 and indicates that the catchment downstream of Gage B contributes proportionally more runoff in larger storm events.

Step 3. Apply Equation B1 to estimate unimpaired flows at Gage B from 1921 to 1952 using observations of unimpaired flow at Gage A over the same time period.

Step 4. From the estimates of unimpaired flows at Gage B made in Step 3, subtract the diversion amount estimated for the time period (Equation B2):

$$\left(IP\ \hat{f}\ at\ B\right) = \left(UIP\ \hat{f}\ at\ B\right) - D\hat{I}V(t) = \hat{\rho}\left(UIP\ f\ at\ A\right)^{\hat{\sigma}} - D\hat{I}V(t) \tag{B2}$$

Here, $\left(IP\ \hat{f}\ at\ B\right)$ is the estimate of impaired flows at gage B [af/month], $D\hat{I}V(t)$ is the estimate of the upstream diversion amount for month *t* (Appendix A), and the other parameters are as defined previously.

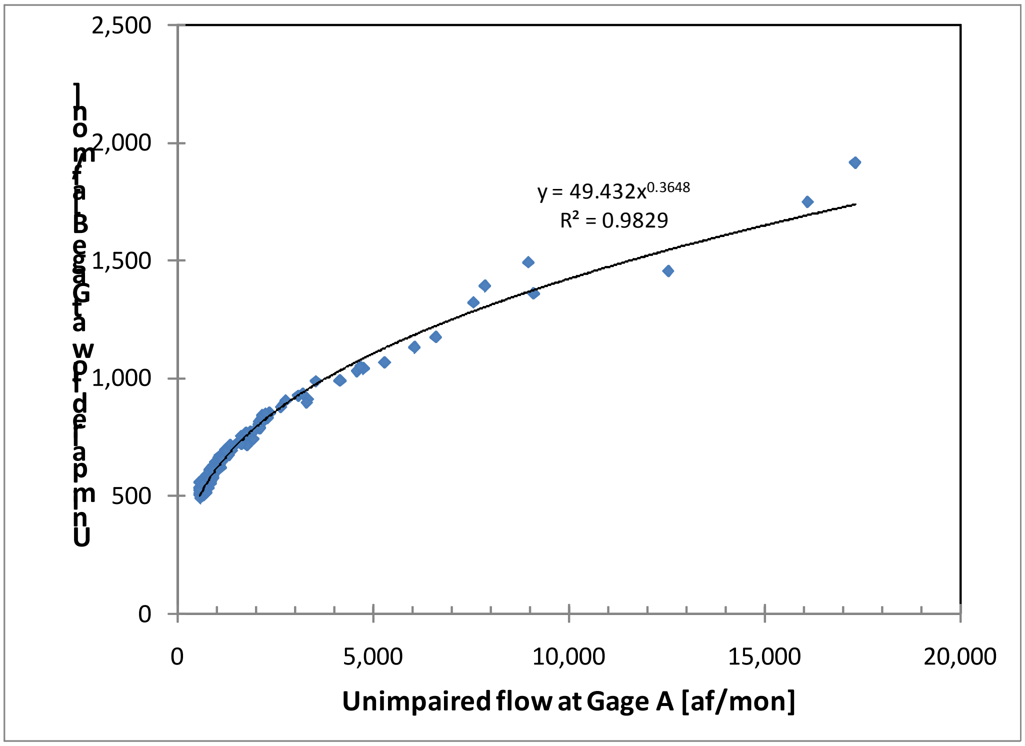

Figure B1. Correlation of unimpaired flows at Gages A and B from 1952 to 1968

Appendix C. Period of record of impaired flow at Gage B

This appendix presents observed and estimated impaired flows at Gage B over the period-of-record from 1921 to 2008 (Table C1). Three methods were used for three different time periods based on the primary and secondary gaged data available. For September, 1968 to September 2008, daily average impaired flow measurements in cfs at Gage B were simply converted to monthly flow volumes in af/month. For January, 1952 to August, 1968, unimpaired flow measurements at Gage B were corrected by an estimated diversion volume as described in Appendix A. And for January, 1921 to December, 1951, unimpaired flow measurements at Gage A were used to estimate unimpaired flows at Gage B which were then corrected for the estimated diversion volume (Appendix B). Observed and estimated unimpaired flows range from slightly above 100 to more than 1,600 af/month.

It was also possible to estimate impaired flows at Gage B between 1952 and 1968 using observations of unimpaired flows at Gage A over the same time period. Comparing these estimates with impaired flows estimated from unimpaired flows at Gage B shows a predominately 1:1 correspondence of results between the two methods with error of up to about 100 af/month (Figure C1).

Table C1. Period of record of estimated unimpaired flow at Gage B (af/mon)

Year	January	February	March	April	May	June	July	August	September	October	November	December
1921	365	369	340	674	265	217	236	207	244	362	357	394
1922	424	511	410	379	191	137	170	222	575	385	442	445
1923	409	459	376	444	296	301	208	535	549	1,567	477	332
1924	575	343	426	332	248	171	499	235	238	360	373	378
1925	608	1,252	326	493	286	200	183	199	255	338	433	392
1926	362	474	387	752	184	160	164	193	229	323	378	467
1927	341	406	358	348	197	491	229	177	457	366	639	393
1928	436	377	454	353	207	180	465	231	204	478	385	885
1929	345	366	395	423	268	634	204	218	209	346	344	529
1930	348	1,066	368	571	288	941	156	252	311	735	376	829
1931	339	396	322	363	228	1,585	1,482	250	224	331	677	369
1932	345	350	339	490	308	145	988	176	233	530	334	1,018
1933	509	448	325	405	237	826	368	219	303	1,439	387	411
1934	394	1,121	961	311	173	226	202	191	242	326	489	876
1935	387	412	323	341	221	170	409	280	209	335	377	335
1936	339	372	297	346	217	215	165	521	171	428	363	362
1937	500	341	350	359	189	207	228	157	280	326	350	418
1938	327	357	392	400	197	169	663	522	240	356	449	625
1939	345	360	382	375	338	195	206	229	450	359	600	546
1940	395	352	334	312	187	541	800	205	1,304	854	486	369
1941	345	366	397	420	814	171	200	417	338	458	2,327	524
1942	422	383	352	389	252	140	171	202	962	634	505	368
1943	329	349	1,036	343	227	502	185	204	180	368	446	515
1944	384	355	640	333	597	168	189	223	208	458	367	470
1945	318	366	569	352	172	154	812	203	160	372	361	377
1946	848	370	329	405	161	181	538	165	496	370	400	419
1947	412	862	378	631	188	338	247	445	180	334	1,447	633
1948	495	506	356	338	1,154	234	224	270	222	1,620	604	363
1949	860	346	2,130	434	156	255	205	172	336	331	574	368
1950	694	536	398	365	207	181	248	190	235	638	398	631
1951	362	349	513	336	309	204	669	203	197	536	400	443
1952	381	660	370	689	407	162	239	363	248	393	362	351
1953	407	649	364	844	272	2,647	788	289	331	406	374	430
1954	367	507	378	345	267	188	359	373	178	453	353	363
1955	477	395	451	460	159	258	224	369	210	369	475	523
1956	479	412	378	375	518	174	307	890	1,368	431	873	374
1957	432	501	353	379	259	288	174	522	374	365	391	366
1958	870	582	653	1,825	175	178	213	243	171	396	377	381
1959	320	857	367	359	183	243	347	371	197	434	398	377
1960	347	362	356	427	221	170	187	271	179	377	340	413
1961	1,429	459	909	345	228	192	486	207	252	331	401	370
1962	400	439	957	336	168	219	798	186	2,756	369	404	349
1963	450	478	546	946	168	230	265	192	253	400	947	347
1964	1,094	524	376	362	223	239	265	329	309	330	454	481
1965	467	354	326	343	397	570	250	454	162	957	361	381
1966	386	484	345	420	819	230	158	185	205	327	348	474
1967	342	370	599	1,111	170	164	297	343	236	474	554	382
1968	425	367	684	405	202	202	236	245	275	475	397	527
1969	644	381	276	353	219	147	170	397	334	346	358	566
1970	323	329	295	345	198	228	241	233	1,034	547	714	305

Table C1. Period of record of estimated unimpaired flow at Gage B (af/mon) (Continued)

Year	January	February	March	April	May	June	July	August	September	October	November	December
1971	403	476	347	379	478	252	281	152	168	383	399	393
1972	352	389	307	474	359	134	317	212	340	426	419	655
1973	331	541	475	414	344	482	185	168	173	443	335	638
1974	309	494	463	458	159	214	299	190	462	940	291	312
1975	468	1,190	335	339	157	376	293	248	258	306	519	733
1976	461	377	485	386	139	403	208	236	250	325	428	430
1977	631	439	671	303	266	281	394	234	291	932	322	440
1978	477	845	566	393	125	156	239	475	275	376	261	706
1979	515	382	500	301	188	222	300	176	120	382	406	437
1980	476	639	411	346	287	218	815	233	277	340	600	911
1981	527	297	970	426	127	190	223	196	296	305	354	516
1982	440	328	409	315	257	324	222	224	260	404	350	322
1983	504	334	310	388	221	873	773	230	148	506	1,026	312
1984	958	317	417	542	315	175	137	242	1,351	359	282	367
1985	560	683	526	394	216	313	566	263	642	365	351	786
1986	401	325	464	368	284	284	340	318	177	464	410	868
1987	540	2,334	1,492	478	179	153	239	493	237	313	418	270
1988	319	393	1,180	257	901	322	156	261	408	526	322	531
1989	434	766	837	436	165	418	161	164	147	671	520	575
1990	635	638	374	317	247	172	311	457	256	300	421	416
1991	593	426	313	286	407	111	457	469	429	352	382	847
1992	383	494	352	489	165	1,478	515	437	193	375	524	380
1993	459	295	524	447	299	231	217	292	1,129	311	368	413
1994	401	529	345	389	185	274	241	204	301	400	455	423
1995	342	396	368	361	373	173	793	160	383	286	504	1,100
1996	377	348	1,129	266	197	256	216	194	269	424	344	593
1997	344	358	652	310	189	196	494	155	202	377	422	323
1998	392	322	519	453	226	160	542	803	183	475	473	433
1999	434	718	309	330	240	367	165	167	176	280	295	874
2000	399	315	293	484	150	221	478	311	175	366	410	289
2001	353	409	282	321	252	417	301	293	188	421	374	402
2002	412	593	484	440	263	385	328	239	219	530	574	327
2003	479	683	386	402	157	366	385	114	299	420	405	376
2004	635	334	469	558	196	244	207	179	212	360	437	276
2005	407	556	396	330	131	391	260	718	432	446	392	312
2006	665	467	1,817	444	240	214	273	251	409	391	312	482
2007	434	1,729	438	382	484	181	340	704	226	397	599	445
2008	398	383	1,502	447	257	676	117	205	216	-	-	-

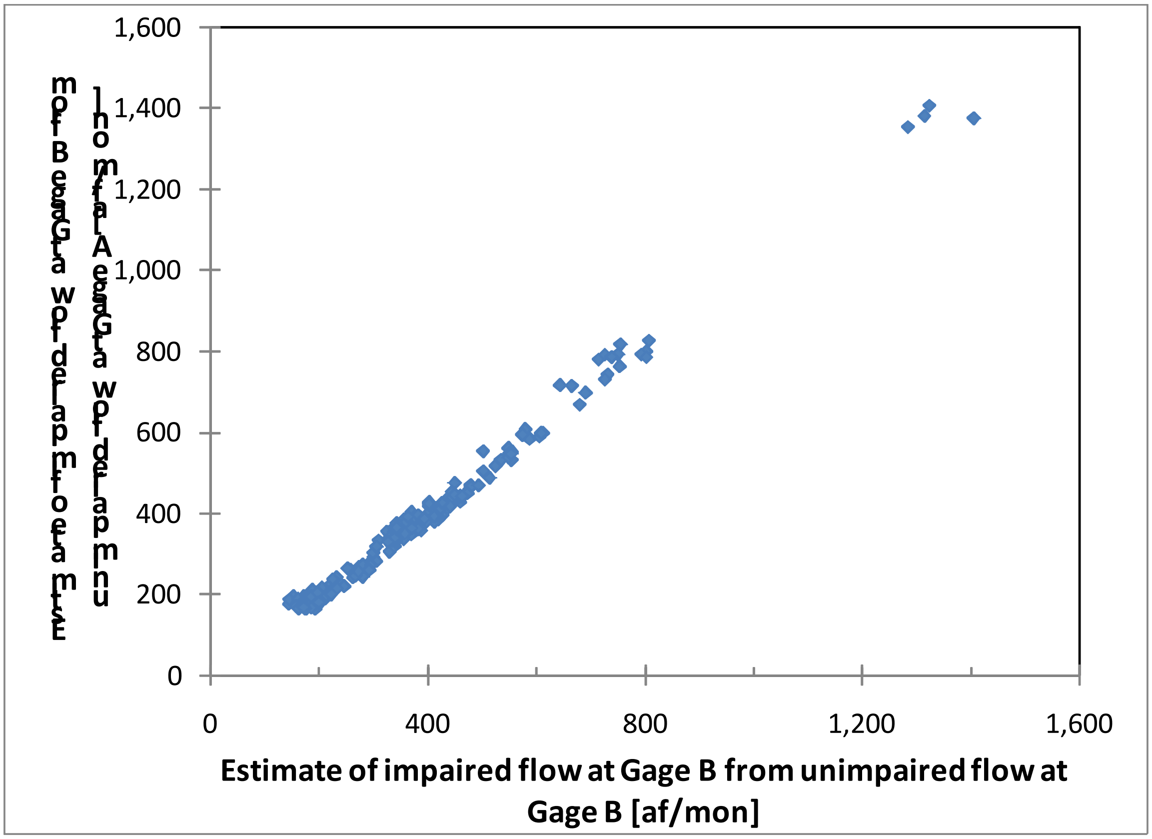

Figure C1. Comparison of impaired flow estimates at Gage B between 1952 and 1968 by the two estimation methods